



# STRUCTURAL SYSTEMS IDENTIFICATION USING WEIGHTED TRANSMISSIBILITY OPERATOR UNDER FORCED VIBRATIONS

*<sup>a</sup> Shanza Arshad Khan\*, <sup>b</sup> Dr. Faheem Butt, <sup>c</sup> Dr. Ahsan Ali*

a: Electrical Engineering Department, UET Taxila, shanza.arshad@students.uettaxila.edu.pk

b: Civil Engineering Department, UET Taxila, faheem.butt@uettaxila.edu.pk

c: Electrical Engineering Department, UET Taxila, ahsan.ali@uettaxila.edu.pk

**Abstract-** The structures are in a continuous state of deterioration due to aging, natural disasters, and other environmental agents, resulting in local or global damages after a specific time. The timely identification of such damages can prevent the huge loss of assets and it can provide an ample opportunity for repair and maintenance. Therefore the service lifespan of the structure can be enhanced by modeling the system and identifying its parameters. Operational modal analysis (OMA) is one of the efficient possible approaches for identifying an unknown, in operation structure and estimating its modal properties like mode shapes, natural frequencies, and damping ratios. Many system identification approaches are being investigated in the literature but most of them are based on time-domain system identification methods which have certain limitations. So there is a need to develop a frequency-based structural systems identification technique that provides clear modal peaks and is more efficient in suppressing fault peaks. In this work, Weighted transmissibility-based operational modal analysis (WTOMA) is applied to a three-floor structure excited by a randomly simulated earthquake. Three sensors are placed on three floors: one of them is attached to the ground floor to record the ground excitations and the remaining two are placed on two floors. The response of the structure in terms of acceleration -time is recorded. The proposed algorithm is then applied to the structural response to obtain its modal parameters.

**Keywords-** Singular values, Loading conditions, Weighted-transmissibility based OMA.

## 1 Introduction

The use of system identification(SI) methods in the civil engineering field has been increasing in recent years. The reason for this is the high success rate of the system identification methods in estimating the mathematical model. It allows civil engineers to make safer designs with the correct estimation of the mathematical model to ensure the integrity of structures. Because system identification plays a major role in reducing the gap between structural systems and their models, also it plays a major role in structural health monitoring(SHM) for damage identification. The system identification in the case of structural systems can be done in the form of obtaining structural response and identifying structural parameters. These parameters could be in the form of vibration signatures, stiffness, stress-strain energies, damping ratios, and mode shapes. In this paper SI will be used for dynamic systems, that is the structural systems that are subjected to dynamic loadings such as seismic, vehicular, wind, or impact loading. OMA is an output-only operational model analysis. It is a field of research that estimates and identifies model properties of operating structures based on output data. This technique assumes that input to the system is white noise sequences which is the only drawback of the classical output-only techniques[1]. OMA techniques are categorized into three different classes; time domain, frequency domain, and time-frequency domain[2] [3] [4] [5].

Transmissibility is defined as a ratio of Fourier transformation of responses located at different points in a system. [6]. Transmissibility-based OMA is an effective approach to identifying model parameters using the output data from ambient or forced vibrations. In the recent past, a new form of transmissibility that is based on power spectrum density



transmissibility (PSDT) is introduced [7] to estimate modal parameters with single loading conditions. The fundamental approaches and applications of transmissibility-based system identification for structural health monitoring are discussed in [8]. A new OMA method based on combining PSDT under different load cases is proposed to enhance the robustness of current PSDT methods with non-white excitations[9].

An efficient approach based on a new concept, that is weighted transmissibility-based OMA (WTOMA) is utilized in this paper to find out and calculate the natural frequencies and mode shapes of a three-story building structure. SVD of the WTOMA matrix and calculation of averaged normalized weighted transmissibility (ANWT) functions are used to extract singular values, mode shapes, and natural frequencies of the system respectively. These functions depict clear modal peaks in the resulting spectra therefore WTOMA is an effective strategy to aid in peak picking [10]. Recently a new time-frequency domain method that combines weighted transmissibility and wavelet transform is proposed to properly identify modal properties of the numerical model under EI Centro earthquake excitation[11]. Another new approach based on deep learning enhanced image processing and PSDT is proposed to realize vibration measurements and identify natural frequencies from PSDT plots[12].

## 2 Research Methodology

### 2.1 Weighted Transmissibility Function

The Transmissibility function is obtained by taking the ratio of the Fourier transform of two output responses of a system at different locations[9].

$$T_{ij}(\omega) = \mathcal{F}\left(\frac{x_i}{x_j}\right) = \frac{X_i}{X_j} = \frac{H_{ik}(\omega)F_k(\omega)}{H_{jk}(\omega)F_k(\omega)} = \frac{H_{ik}(\omega)}{H_{jk}(\omega)} \quad (1)$$

In Weighted transmissibility, the transmissibility function gets divided by its absolute or mod value raised to some power gamma  $\gamma$ . Therefore the weighted transmissibility function is calculated as:

$$WT_{ij}(\omega) = \frac{T_{ij}(\omega)}{|T_{ij}(\omega)|^\gamma} \quad (2)$$

Whereas the weight function is given by:

$$W_{ij}(\omega) = \frac{1}{|T_{ij}(\omega)|^\gamma} \quad (3)$$

The parameter gamma “  $\gamma$  ” is chosen based on the required accuracy level. This parameter ‘ $\gamma$ ’ indicates the extent of difference that is allowed between the columns of the weighted transmissibility matrix. The weighted transmissibility matrix is given below:

$$WTM(\omega) = \begin{bmatrix} WT_{1j}^1(\omega) \cdots WT_{1j}^p(\omega) & WT_{1j}^{p+1}(\omega) \cdots WT_{1j}^N(\omega) \\ \vdots & \vdots \\ WT_{ij}^1(\omega) \cdots WT_{ij}^p(\omega) & WT_{ij}^{p+1}(\omega) \cdots WT_{ij}^N(\omega) \\ \vdots & \vdots \\ WT_{Nj}^1(\omega) \cdots WT_{Nj}^p(\omega) & WT_{Nj}^{p+1}(\omega) \cdots WT_{Nj}^N(\omega) \end{bmatrix} \quad (4)$$

here ‘j’ indicates a fixed reference DOF.

### 2.2 Weighted Transmissibility Based OMA

In this paper, a very efficient approach is investigated for the identification of the modal properties of any structure using the weighted transmissibility operator[9]. The decomposition of the weighted transmissibility matrix can be done using the SVD technique as follows[10].

$$[WTM(\omega)] = [U(\omega)][S(\omega)][V(\omega)]^H \quad (5)$$



The  $[V(\omega)]$  and  $[U(\omega)]$  are the right and left singular vectors of the transmissibility matrix. The matrix  $[S(\omega)]$  is a diagonal matrix consisting of the singular values with  $s_1(\omega) \geq s_2(\omega) \geq \dots \geq s_l(\omega)$  where “H” is representing the conjugate transpose.

By calculating the product of inverses of second singular values with different degrees of freedom obtained from  $WTM(\omega)$  can be used to calculate PIS ( $\omega$ ) function as follows:

$$PIS(\omega) = \prod_{j=1}^N \left( \frac{1}{S_2} \right)_j \quad (6)$$

Along with this, the mode shapes can also be derived by calculating the product between entities of left-most singular vectors  $[U_{1ij}(\omega)]$  and the function  $|T_{ij}(\omega)|^Y$  which is the inverse of the weight function  $W_{ij}(\omega)$  as follows:

$$\Phi_{rij} = U_{1ij}(\omega) \frac{1}{N} \sum_{p=1}^N |T_{ij}(\omega)|^Y \quad (7)$$

### 3 Flowchart of the Algorithm:

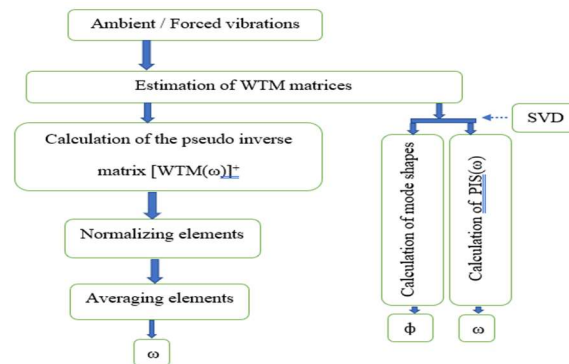


Figure 1: Flowchart of the WTOMA Algorithm

### 4 Experimental Procedures

The structure chosen for the current study is shown in Figure 2. The structure consists of three floors. The total length/width of the structure is 3600mm, the total height excluding footing is 7356mm, whereas each slab is 144 mm thick (Table 2). The total weight of the structure is 19.873 tons.

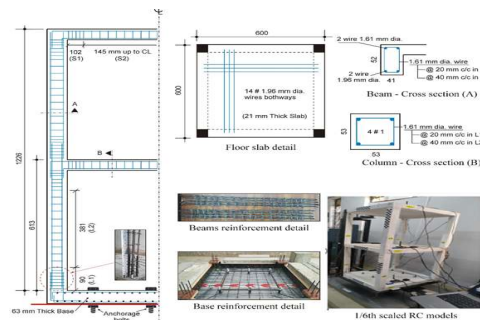


Figure 2: The prototype structure and its scaled model



Whereas the model used for analysis is obtained by scaling the structure and hence reduced model is obtained with a length/width of 600mm, a height of 1226mm, slab thickness of 23mm, and a total weight of 0.268tons. The structural scaling is performed by choosing a scale factor for each of the structure parameters and hence corresponding reduced model is shown in Table 1.

Table 1: Prototype Structural scaling details

Parameter	Scale Factor	Prototype Structure	Model (Provided)
Concrete	1	24 MPa	*Micro Concrete
Steel Reinforcement	1	Grade 60	*Grade 60
Column	Cross Section Area	36	900 cm <sup>2</sup>
	Rebar Diameter	6	19 mm
	Confinement Bar Diameter	6	10 mm
Beam 1 <sup>st</sup> Floor (B1)	Cross Section Area	36	690 cm <sup>2</sup>
	Bottom Rebar Diameter	6	19 mm
	Top Rebar Diameter	6	13 mm
	Confinement Bar Diameter	6	10 mm
Beam 2 <sup>nd</sup> Floor (B2)	Cross Section Area	36	690 cm <sup>2</sup>
	Bottom Rebar Diameter	6	13 mm
	Top Rebar Diameter	6	13 mm
	Confinement Bar Diameter	6	10 mm
Slab	6	10 mm	1.61 mm

. The dimensional details of both the prototype structure and its model are shown in Table 2.

Table 2: dimensional details of the prototype structure and its model

No.	Parameter	Prototype Structure	Model
1	Length/Width	3600 mm	600 mm
2	Height excluding footing	7356 mm	1226 mm
3	Column Size	300 mm × 300 mm	53 mm × 53 mm
4	Beam Size	230 mm × 300 mm	41 mm × 52 mm
5	Tie Beam	230 mm × 380 mm	63 mm thick footing pad
6	Slab Thickness	140 mm	23 mm
8	Weight	19.873 Tons	0.1239 Tons

## 5 Results

The results obtained through the investigated algorithm are shown below. The structure is excited by a randomly simulated earthquake. The response of the building structure i.e. acceleration – time history of the three sensors during the period of 100s is shown in Figure 3.

### 5.1 Use of Software

The software used for identifying and estimating the parameters of the structure is MATLAB.

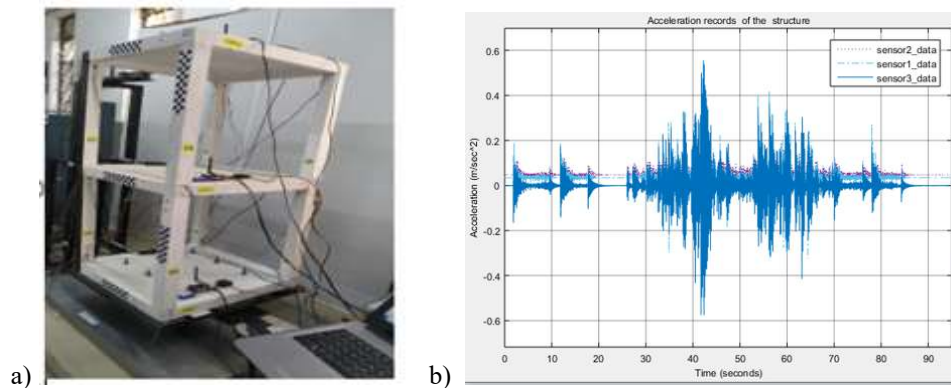


Figure 3: a. Building structure prototype with installed sensors b. sensors response time history

The first two natural frequencies obtained through the algorithm and the mode shapes corresponding to the two estimated natural frequencies are shown in Figure 4 and Figure 5.

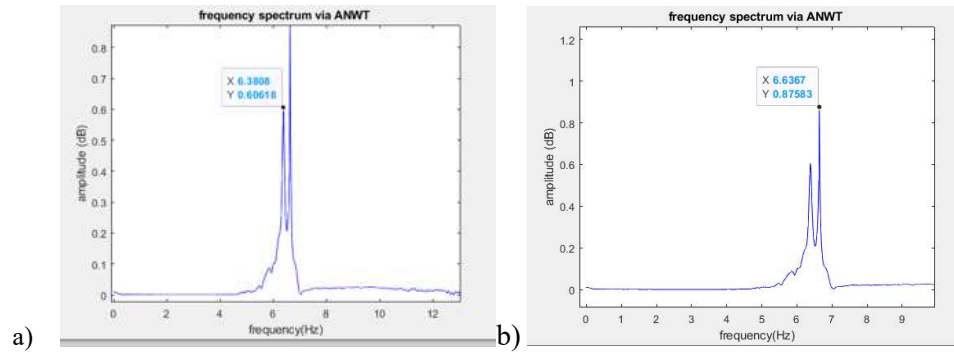


Figure 4: The natural frequencies obtained from WTOMA a. 1st natural frequency b. 2nd natural frequency

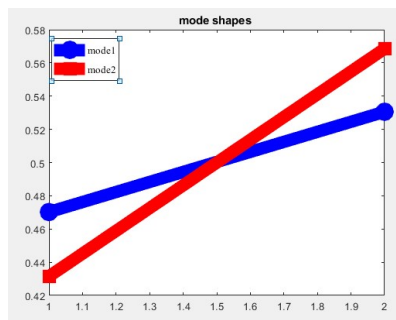


Figure 5: The mode shapes corresponding to the two frequencies

## 5.2 Comparison Of Results :

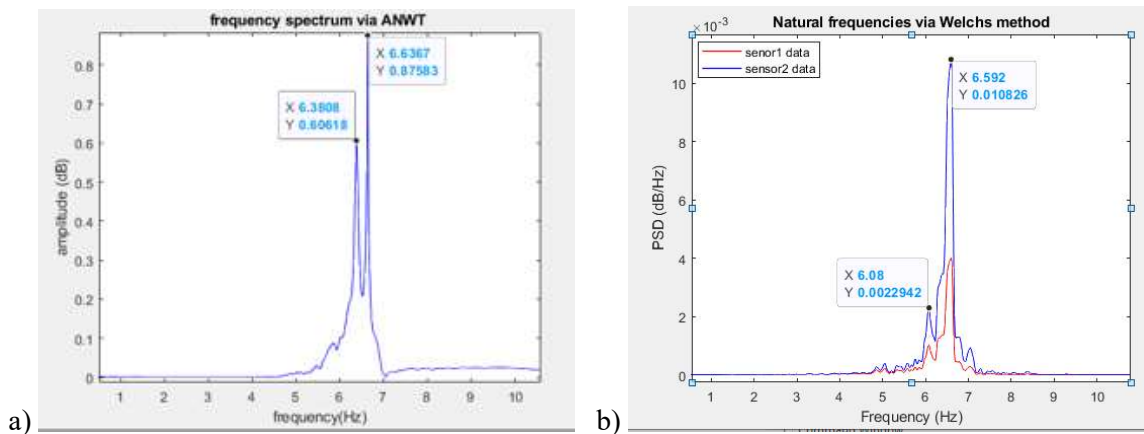


Figure 6: Comparison of a. WTOMA and b. Welch's averaged PSD

The results are compared with Welch's averaged PSD estimates, the frequencies obtained through the WTOMA algorithm are 6.38084Hz and 6.6367Hz respectively while the frequencies obtained from Welch's averaged are 6.08Hz and 6.592Hz respectively. As it can be seen that Welch's method suffers from noise sensitivity hence many false peaks are seen in the PSD plot. Whereas the WTOMA algorithm is more effective in displaying spectral peaks with strength and clarity. Along with this false peaks are minimized to a much extent and there is less spectral leakage as compared to Welch's averaged PSD.



## 6 Utilization Of Research Results:

The method can be used for identifying and estimating model parameters in real-time in operating systems such as bridges, buildings, and other engineering structures. The outcome of this research can be used for the prediction of structural failures and damage detection in the field of SHM.

## 7 Conclusion

The following conclusions can be drawn from the study conducted in this paper:

1. Spectral leakage is minimized hence strong and clear peaks occur in the resulting spectra.
2. Reduced noise sensitivity hence fewer false peaks.
3. Clearer mode shapes are obtained.

The results demonstrate that WTOMA is more effective in identifying modal parameters of a structure as it aids in the peak picking process hence strong spectral peaks occur in the resulting spectra. False peaks are minimized to a much extent and clearer mode shapes are obtained. Whereas Welch's method is more sensitive to noise and the concept of windowing function creates spectral leakages. Lower model order results in smoothing while higher order induces false peaks in Welch's method.

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